Robust optimization for interactive multiobjective programming with imprecise information applied to R&D project portfolio selection

By: Farhad Hassanzadeh, Hamid Nemati, Minghe Sun

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Abstract:

A multiobjective binary integer programming model for R&D project portfolio selection with competing objectives is developed when problem coefficients in both objective functions and constraints are uncertain. Robust optimization is used in dealing with uncertainty while an interactive procedure is used in making tradeoffs among the multiple objectives. Robust nondominated solutions are generated by solving the linearized counterpart of the robust augmented weighted Tchebycheff programs. A decision maker's most preferred solution is identified in the interactive robust weighted Tchebycheff procedure by progressively eliciting and incorporating the decision maker's preference information into the solution process. An example is presented to illustrate the solution approach and performance. The developed approach can also be applied to general multiobjective mixed integer programming problems.

Keywords: Multiobjective programming | Robust optimization | Imprecise information | Portfolio selection | Interactive procedures

Article:

1. Introduction

In today's fast paced and highly competitive economy, engaging in meaningful Research and Development (R&D) activities is essential for any organization striving to achieve and maintain competiveness. R&D projects are resource intensive and, therefore, benefits gained from and costs associated with each R&D project must be carefully considered. R&D project portfolio selection is a complex and non-trivial problem with important organizational implications.

Organizations usually have more candidate R&D projects than they have resources to support them. The purpose of R&D project portfolio selection is to select a feasible subset of promising projects as a portfolio from a set of candidate projects based on multiple criteria. R&D project portfolio selection is always constrained by limited resources such as budget, research staff, laboratory space, and other technical scarcities. In addition, R&D project portfolio selection may have other restrictions such as corporate policies and contractual relationships with other stakeholders. Furthermore, uncertainties are always involved in R&D, such as uncertainties in the outcomes of the projects, in the resource availability and usage, and in the interdependence and interactions among the projects. Given these constraints and uncertainties, R&D managers must select a portfolio of projects based on multiple criteria representing corporate goals or objectives. Objectives, such as profit maximization, market share maximization, risk minimization, or human resource utilization minimization, are usually conflicting and fraught with uncertainties which further complicate the R&D project portfolio selection. The challenge is how to select the best portfolio of R&D projects based on these competing objectives within the resource restrictions while giving consideration to uncertainties.

R&D is often an original endeavor with long lead time and unclear life time expenditure, resource usage and market outcome. These unique characteristics imply that much of the information required in making R&D decisions is very imprecise and impossible to accurately estimate. To address uncertainties, probabilistic and fuzzy approaches have been proposed to capture the imprecision of data by considering reasonable distributions to describe possible values of imprecise coefficients in optimization models. One drawback of such approaches is, however, that they cannot handle the situation where there is a possible range for each of these coefficients, but the most probable or plausible value within the range cannot be estimated (Carlsson, Fullér, Heikkilä, & Majlender, 2007). This calls for novel approaches which can more adequately capture the real-world situation of R&D project portfolio selection.

The focus of this study is to develop a method for dealing with imprecise information associated with the multiobjective problem of selecting a portfolio of R&D projects. The proposed method integrates two complementary approaches to deal with both uncertainties and multiple objectives. Uncertainties in the problem coefficients, both in the objective functions and constraints, are modeled through robust optimization while the multiobjective problem is solved through interactive multiobjective programming. Interval uncertainties, *i.e.*, each imprecise coefficient belongs to an interval of real numbers without prior distribution details, are assumed. An interactive approach is used to capture the decision maker (DM)'s preference information with respect to the multiple objectives in the problem.

The remainder of the paper is organized as follows. The relevant R&D project portfolio selection and robust optimization literature is reviewed in Section 2. The nominal multiobjective binary integer programming model for R&D project portfolio selection is presented in Section 3. Following a brief introduction to robust optimization, the robust counterpart of the nominal model is formulated in Section 4. The solution of the robust counterpart within an interactive procedure by repeatedly solving robust augmented weighted Tchebycheff programs is discussed in Section 5. An example of R&D project portfolio selection problem is presented to illustrate the proposed approach and the results of computational experimentation are reported in Section 6. Finally, the article concludes with a summary in Section 7.

2. Previous work

Portfolio selection, whether financial, investment or R&D, is always fraught with uncertainty and is inherently multiobjective. Steuer, Qi, and Hirschberger (2005) presented a list of possible objectives in a financial portfolio selection problem. Because the objectives are incommensurate, the DM's preference information has to be used to make tradeoffs in order to find a final portfolio. However, a review of the literature in this area indicates that most of proposed solution techniques either focus on the multiple objectives or address the uncertainties but not both.

Multiobjective optimization techniques in solving the multiobjective project portfolio selection problem, like in other applications, can be classified into three major categories, *i.e.*, requiring *a priori*, *a posteriori*, and progressive articulation of preference information, based on the time the DM's preference information is articulated and used (Hwang & Masud, 1979).

In the first category, *a priori* preference information articulation from the DM regarding the criteria is assumed, and a compromise solution is obtained by converting multiple objectives of the problem to a single objective. To this end, some authors assign different weights to the objective functions according to their importance to the DM, and use a weighted sum of the objective functions as a single objective function (Ghasemzadeh et al., 1999, Klapka and Piños, 2002 and Medaglia et al., 2008). This approach can only find basic solutions for linear problems and may fail to balance objective functions in relation to their importance (Steuer, 1986). Some authors use goal programming to address this problem (Badri et al., 2001, Graves and Ringuest, 1992, Lee and Kim, 2001, Santhanam and Kyparisis, 1995, Schniederjans and Santhanam, 1993 and Zanakis et al., 1995). Azmi and Tamiz (2010) provided a review of the goal programming approaches. However, setting aspiration levels and weights for the goals is challenging and may even result in a dominated solution (Ringuest and Graves, 1989 and Santhanam and Kyparisis, 1995).

The second category includes approaches requiring *a posteriori* articulation of DM's preference information. Accordingly, it is assumed that *a priori* preference information articulation regarding the criteria is unavailable. Therefore, a two-phase procedure is implemented that first identifies the whole or a large set of efficient, *i.e.*, Pareto-optimal or nondominated, portfolios possibly using metaheuristics (Carazo et al., 2010, Doerner et al., 2004, Doerner et al., 2006, Ghorbani and Rabbani, 2009, Rabbani et al., 2010,Stummer and Sun, 2005 and Yu et al., 2012), and then explores the set of identified efficient solutions possibly through an interactive approach (Stummer and Heidenberger, 2001 and Stummer and Heidenberger, 2003). However, determining the set of all efficient solutions is challenging and becomes increasingly demanding or even impossible as the number of projects and/or the number of objectives grows because integer programming problems are usually NP hard. In addition, the DM may be confronted with a large number of competing portfolios in the second phase and selecting the one that is most preferred is not an easy task, which further complicates the process of project portfolio selection.

The third category includes interactive approaches in which the DM's preference information is progressively articulated during, and incorporated into, the solution process so as to locate the DM's most preferred solution. Interactive methods are one of the most promising approaches for solving multiobjective programming problems (Steuer, 1986). Zopounidis, Despotis, and Kamaratou (1998) developed a multiobjective linear programming model to select a portfolio of stocks and used an interactive approach to solve the problem. Stummer and Heidenberger, 2001 and Stummer and Heidenberger, 2003presented an interactive procedure for solving the multiobjective R&D project portfolio selection problems. Steuer et al. (2005) discussed tools and techniques from multiple criteria optimization to analyze and solve the portfolio selection problem.

The majority of project portfolio selection formulations in the literature are based on deterministic data. However, as mentioned earlier, an important characteristic of R&D project portfolio selection is that future attributes of R&D projects, *e.g.*, costs and revenues, availability and usage of human resources and material supplies, development of technical skills and risks, and market outcomes, are very difficult to estimate. Consequently, stochastic (Abdelaziz et al., 2007, Birge and Louveaux, 1977, Gabriel et al., 2006, Gutjahr and Reiter, 2010 and Medaglia et al., 2007) and fuzzy (Aryanezhad et al., 2011,Bhattacharyya and Kar, 2011, Coffin and Taylor, 1996, Łapuńka, 2012 and Tolga, 2008) approaches are introduced to the classical multiobjective programming formulations to address the issue of incomplete and imprecise information. However, both of these approaches assume prior details about coefficient distributions, an assumption which is often flawed for R&D projects as ground-breaking endeavors.

There are a few studies that address uncertainties of project portfolio selection within an interactive procedure. Nowak (2006) developed an interactive procedure for selecting one project that is based on STEM (Benayoun, de Montgolfier, Tergny, & Laritchev, 1971), a well-known interactive procedure for multiobjective programming. In this procedure, risk and uncertainty are modeled through stochastic dominance. Shing and Nagasawa (1999) proposed an interactive portfolio selection method for selecting a preferred portfolio from a set of candidate portfolios for the case where the mean and the variance of returns of securities have several scenarios with known occurrence probabilities.

Robust optimization is a relatively new approach that has experienced an explosion of applications in many areas of management science such as supply chain management, health care systems, and portfolio selection (Gabrel, Murat, & Thiele, 2013). In robust optimization, imprecise information is incorporated by way of set inclusion, *i.e*., the true value of an imprecise coefficient is contained in an interval characterized by the DM without any assumption on the distribution of the coefficient. Introduced by Soyster (1973), robust optimization addresses the problem of data uncertainty by guaranteeing the feasibility and optimality of the solution for the worst instances of the problem, which results in overly conservative solutions. Ben-Tal and Nemirovski (2002) and El-Ghaoui, Oustry, and Lebret (1998) took first steps to address the over conservatism issue and considered ellipsoidal uncertainties, which results in conic quadratic robust counterparts for linear formulations with uncertainties. Although ellipsoidal uncertainties can be used to approximate more complicated uncertainty sets, they lead to counterpart models that are nonlinear and, hence, computationally less tractable and less practical than linear models. Bertsimas and Sim, 2003 and Bertsimas and Sim, 2004 developed the "budget of uncertainty" approach leading to robust counterparts that have the advantage of being linear rather than quadratic. In addition, their approach provides full control over the degree of conservatism for every constraint.

A recent literature review by Gabrel et al. (2013) reveals that almost all of the studies that have introduced robust formulations of multiobjective problems have used notions of robustness that are very different from the classical robust optimization concepts, resulting in complex optimization models that can only be tackled by heuristic or metaheuristic methods (Chen et al., 2012, Deb and Gupta, 2005, Düzgün and Thiele, 2010, Gaspar-Cunha and Covas, 2008, Laguna, 1995, Liesiö et al., 2008, Luo and Zheng, 2008,Ono et al., 2009 and Suh and Lee, 2001). Perhaps the only relevant work in multiobjective robust optimization is by Hu and Mehrotra (2012) where objective function weights are uncertain and all other model parameters are deterministic. They used a multicriteria robust weighted sum approach called McRow to identify a robust Pareto optimal solution that minimizes the worst-case weighted sum of the objectives.

As a major step forward, the current study develops an interactive robust optimization procedure based on the budget of uncertainty approach (Bertsimas and Sim, 2003 and Bertsimas and Sim, 2004) to solve multiobjective R&D portfolio selection problems with imprecise coefficients in the objective functions and constraints. The approach developed in this study is the first method that truly extends the classical robust optimization concept to multiobjective optimization and opens a new avenue for solving other multiobjective optimization problems under uncertainties.

3. Problem statement

The aim of the multiobjective R&D project portfolio selection problem is to select a subset as a portfolio from a large set of possible candidate projects considering multiple conflicting objectives, subjected to a set of constraints. Let K denote the number of objective functions, m the number of constraints, and n the number of candidate projects in the entire set.

There is no prior requirement for the number of projects to be selected into the portfolio. Without loss of generality, all objective functions are assumed to be minimized. The multiobjective R&D project portfolio selection model is stated as in (1) in the following:

equation(1)

$$\begin{aligned} \min \quad & z_k = f_k(\mathbf{x}) \quad \forall k \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leqslant b_i \quad \forall i \\ & \mathbf{x} \in \mathbb{B}^n. \end{aligned}$$

In the model, $\mathbf{x}$ is the vector of binary decision variables, $f_k(\mathbf{x}) = \sum_{j=1}^{n} c_{kj} x_j$ is the k th objective function, and $g_i(\mathbf{x}) = \sum_{j=1}^{n} a_{ij} x_j \leqslant b_i$ is the ith constraint. Although each application is different, the objective functions may include the maximization of total expected profit, maximization of expected market share, or minimization of total expected risk, while the constraints may include limited budget, scarce human and material resources, and interdependence and interaction among the candidate projects. A project j is selected into the portfolio if $x_j = 1$ and otherwise if $x_j = 0$. In addition to the binary decision variables representing the selection of projects, other decision variables may be used in the model to represent the interdependencies and interactions among the projects in some specific applications. The multiobjective R&D project portfolio selection model (1) is the nominal model assuming the values of a_{ij}, $\forall i, j$, and c_{kj}, $\forall k, j$, are exactly known. The nominal model (1) is an ordinary multiobjective binary integer programming model.

Since the objective functions are usually in conflict, model (1) usually does not have a single feasible solution that simultaneously minimizes all K objective functions. The optimal solution is defined to be a feasible solution that maximizes the DM's value function (Steuer, 1986 and Yu, 1985). Because the DM's value function is not readily available, the solution process of the multiobjective R&D project portfolio selection model (1) is to search for a solution which is most preferred by the DM.

The following concepts are borrowed from Steuer (1986). The set of solutions satisfying all constraints, *i.e.* , $X=\{\mathbf{x}\in B^n | g_i(\mathbf{x}) \leqslant b_i, \forall i\}$, is the feasible region, and a point $\mathbf{x}\in X$ is a feasible solution, in decision space. The set $Z=\{\mathbf{z}\in R^K | z_k = f_k(\mathbf{x}), \mathbf{x}\in X\}$ is the feasible region in criterion space. A point $\mathbf{z}\in Z$ is a feasible solution in criterion space or a feasible criterion vector. A point $\bar{\mathbf{z}} \in Z$ is a nondominated criterion vector if there does not exist any criterion vector $\mathbf{z}\in Z$, such that $\mathbf{z} \leqslant \bar{\mathbf{z}}$ and $\mathbf{z}\neq\bar{\mathbf{z}}$. $\overline{Z}$ is used to represent the set of all nondominated solutions in criterion space. A point $\bar{\mathbf{x}} \in X$ is an efficient solution in decision space if $\bar{\mathbf{z}} \in \overline{Z}$ such that $\bar{z}_k = f_k(\bar{\mathbf{x}})$, $\forall k$. $\overline{X}$ is used to represent the set of all efficient solutions in decision space. A criterion vector $\hat{\mathbf{z}} \in Z$ is optimal if it maximizes the DM's value function. However, a DM's value function in real-life problems is hard to estimate and its functional form is usually unknown (Yu, 1985). If $\hat{\mathbf{z}}$ is optimal, $\hat{\mathbf{z}} \in \overline{Z}$, *i.e* ., an optimal solution must be nondominated. A point $\mathbf{z}^* \in R^K$, such that $z_k^* = \min\{f_k(\mathbf{x}), \mathbf{x} \in X\}, \forall k$, is the ideal point. For most multiobjective programming problems, $\mathbf{z}^* \notin Z$,*i.e.*, $\mathbf{z}^*$ is infeasible. A point $\mathbf{x}^* \in B^n$ such that $z_k^* = f_k(\mathbf{x}^*), \forall k$, usually does not exist (Sun, 2005). A point$\mathbf{z}^{**} \in R^K$, such that $z_k^{**} = z_k^* - \varepsilon_k$, where $\varepsilon_k > 0$ and small $\forall k$, is called a utopian point.

When a multiobjective programming problem is solved, especially when an interactive procedure or an approach requiring *a posteriori* articulation of the DM's preference information is used, many nondominated solutions need to be generated as trial solutions. These nondominated solutions are usually evaluated by the DM so as to elicit preference information from the DM. Nondominated solutions are usually generated by solving augmented weighted Tchebycheff programs derived from the nominal model(1) (Steuer, 1986). The weighting vector space is defined as

equation(2)

$$\mathbf{W} = \left\{ \mathbf{w} \in \Re^K \middle| w_k > 0, \sum_{k=1}^{K} w_k = 1 \right\}.$$

Any $\mathbf{w} \in \mathbf{W}$ is a weighting vector. For a given $\mathbf{w} \in \mathbf{W}$, an augmented weighted Tchebycheff program for the nominal model (1) is formulated as in (3) in the following

equation(3)

$$\begin{aligned} \min \quad & \alpha + \rho \sum_{k=1}^{K} (z_k - z_k^{**}) \\ \text{s.t.} \quad & \alpha \geqslant w_k (z_k - z_k^{**}) \quad \forall k \\ & z_k = f_k(\mathbf{x}) \qquad \forall k \\ & g_i(\mathbf{x}) \leqslant b_i \qquad \forall i \\ & x_j \in \{0, 1\} \qquad \forall j \\ & z_k \text{ unrestricted} \quad \forall k \\ & \alpha \geqslant 0, \end{aligned}$$

where $\rho > 0$ is a small scalar. Usually $\rho = 0.001$ is sufficient.

Note that in the augmented weighted Tchebycheff program (3), each objective function is converted into a constraint and, hence, the number of objective functions is not a concern from a computational point of view. The augmented weighted Tchebycheff program (3) is a single objective binary integer programming problem. If its optimal solution is represented by the composite vector ($\mathbf{x}_w$, $\mathbf{z}_w$, α_w), then $\mathbf{x}_\mathbf{w} \in \overline{X}$ and $\mathbf{z}_\mathbf{w} \in \overline{Z}$, *i.e.*, $\mathbf{x}_w$ is efficient and $\mathbf{z}_w$ is nondominated. For a given $\mathbf{w} \in \mathbf{W}$, the augmented weighted Tchebycheff program (3) generates a given nondominated solution. By using a widely dispersed set of weighting vectors in **W**, a widely dispersed set of representative nondominated solutions can be generated.

As previously stated, the values of most of the coefficients in the nominal model (1) are not known with certainty given the nature of the R&D project portfolio selection problem. When the values of these coefficients are not precisely known, the solution obtained for the nominal model (1) may not be close to the true most preferred solution of the DM or, even worse, could be infeasible for a realization of these imprecise coefficients. Given that a_{ij}, $\forall i, j$, and c_{kj}, $\forall k, j$, are uncertain and their exact values are unknown but within a certain interval, the focus of this study is on finding a solution of model (1) such that the solution is not only feasible with a very high probability, but also very close to the most preferred solution of the DM. An interactive robust weighted Tchebycheff procedure is proposed for this purpose.

4. Robust optimization for R&D project portfolio selection

A robust optimization framework is briefly discussed first for single objective optimization problems. This framework is then extended to multiobjective optimization problems through the augmented weighted Tchebycheff program (3).

4.1. The robust optimization framework for single objective problems

Consider the following standard binary integer programming problem

equation(4)

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leqslant b_i \quad \forall i \\ & \mathbf{x} \in \mathbb{B}^n \end{aligned}$$

where $g_i(\mathbf{x}) = \sum_{j=1}^{n} a_{ij} x_j \leqslant b_i$, as in (1), is the i th constraint and $f(\mathbf{x}) = \sum_{j=1}^{n} c_j x_j$ is the single objective function of the problem. Model (4) with precise a_{ij}, $\forall i, j$, is the nominal formulation.

Now assume that each a_{ij} is an imprecise coefficient with unknown exact value in the interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$ where $\bar{a}_{ij}$ is the nominal value and $\hat{a}_{ij}$ is the half-interval width of a_{ij}. The precise values of c_j, $\forall j$, are assumed to be known. The purpose of robust optimization is to find an optimal solution, called the robust optimal solution, which remains feasible for almost all possible realizations of the imprecise problem coefficients. Obviously, as much as it is unlikely that all uncertain coefficients are equal to their nominal values, it is also unlikely that they are all equal to their worst-case values. The worst-case solution actually occurs with a negligible probability because large deviations in the coefficients a_{ij} tend to cancel out each other as n grows. Consequently, the most conservative approach, where all coefficients are equal to their worst-case values, leads to a severe deterioration of the optimal solution without being fairly justified in practice. Therefore, adjusting the degree of conservatism of the solution in order to make a reasonable trade-off between robustness and performance is a necessity (Bertsimas and Sim, 2003 and Bertsimas and Sim, 2004).

This concept is quantified by reformulating the nominal model in (4). The absolute value of the scaled deviation of the imprecise coefficient a_{ij} from its nominal value $\bar{a}_{ij}$, denoted by δ_{ij}, is defined in the following

equation(5)

$$\delta_{ij} = |(a_{ij} - \bar{a}_{ij})/\hat{a}_{ij}| \quad \forall i, j.$$

Apparently, δ_{ij} takes values in the interval [0, 1]. A budget of uncertainty Γ_i is imposed to the ith constraint in the following sense

equation(6)

$$\sum_{j=1}^{n} \delta_{ij} \leqslant \Gamma_i \qquad 0 \leqslant \Gamma_i \leqslant n,$$

where $\Gamma_i = 0$ and $\Gamma_i = n$ correspond to the nominal and worst cases, respectively. Bertsimas and Sim, 2003 and Bertsimas and Sim, 2004 showed that varying Γ_i in the interval [0, n] will appropriately adjust performance against robustness. Intuitively, the use of Γ_i can rule out large deviations in $\sum_{j=1}^{n} a_{ij} x_j$ that play a predominant role in worst-case analysis but happens with negligible probability. When each a_{ij} is treated as a variable, the nonlinear robust formulation of the nominal model in (4) can be stated as (7) in the following

equation(7)

$$\begin{array}{ll} \min & f(\mathbf{x}) \\ \text{s.t.} & \max\limits_{\mathbf{a}_i} \left[\bar{g}_i(\mathbf{a}_i, \mathbf{x}) \middle| \sum_{j=1}^{n} \delta_{ij} \leqslant \Gamma_i \right] \leqslant b_i \quad \forall i \\ & \mathbf{x} \in \mathbb{B}^n, \end{array}$$

where $\mathbf{a}_i$ is the vector of imprecise coefficients in the i th constraint with each $a_{ij} \in [\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$ and $\bar{g}_i(\mathbf{a}_i, \mathbf{x}) = \sum_{j=1}^{n} a_{ij} x_j$ is the counterpart of $g_i(\mathbf{x})$ in (4) but with each a_{ij} treated as a variable. Bertsimas and Sim (2003) proved that the nonlinear robust formulation in (7) has the following robust linear counterpart

equation(8)

$$\begin{aligned}
\min \quad & \sum_{j=1}^{n} c_j x_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} \bar{a}_{ij} x_j + \Gamma_i q_i + \sum_{j=1}^{n} r_{ij} \leqslant b_i && \forall i \\
& q_i + r_{ij} \geqslant \hat{a}_{ij} y_j && \forall i, j \\
& -y_j \leqslant x_j \leqslant y_j && \forall j \\
& q_i \geqslant 0 && \forall i \\
& y_j \geqslant 0, x_j \in \{0, 1\} && \forall j \\
& r_{ij} \geqslant 0 && \forall i, j.
\end{aligned}$$

The highly attractive feature of this formulation is that the counterpart model is of the same class and complexity as the nominal model in (4). In addition, even if the budget of uncertainty constraints are not satisfied, the robust solution will remain feasible with a very high probability (Bertsimas and Sim, 2003 and Bertsimas and Sim, 2004).

4.2. Application to multiobjective problems

Under uncertainty, the problem coefficients in (1) are uncertain and, hence, the selected portfolio must be robust, *i.e.*, the solution should remain feasible (constraint robust), efficient and most preferred by the DM (objective function robust) under almost all possible realizations of imprecise coefficients. Both a_{ij} and c_{kj} are considered uncertain and their uncertainty is captured using the interval uncertainty discussed earlier. The nominal value $\bar{a}_{ij}$ and the half-interval width $\hat{a}_{ij}$ of each a_{ij} are defined in the same way as in the single objective optimization problem discussed above. The nominal value and the half-interval width of c_{kj} are represented by $\bar{c}_{kj}$ and $\hat{c}_{kj}$, respectively. The k th objective function is expressed as $\bar{f}_k(\mathbf{c}_k, \mathbf{x}) = \sum_{j=1}^{n} c_{kj} x_j$, where $\mathbf{c}_k$ is the vector of imprecise coefficients in the k th objective function with each $c_{kj} \in [\bar{c}_{kj} - \hat{c}_{kj}, \bar{c}_{kj} + \hat{c}_{kj}]$. While being the counterpart of $f_k(\mathbf{x})$ in (1), $\bar{f}_k(\mathbf{c}_k, \mathbf{x})$ is a function of both $\mathbf{c}_k$ and $\mathbf{x}$, because each c_{kj} is treated as a variable. Similar to δ_{ij} defined for a_{ij} in (5), the absolute value of the scaled deviation of c_{kj} from its nominal value $\bar{c}_{kj}$, denoted by δ'_{kj}, is defined in the following:

equation(9)

$$\delta'_{kj} = |(c_{kj} - \bar{c}_{kj})/\hat{c}_{kj}| \quad \forall k, j.$$

Similar to (6), a budget of uncertainty Γ'_k is imposed to the kth objective function such that

equation(10)

$$\sum_{j=1}^{n}\delta'_{kj} \leqslant \Gamma'_k \qquad 0 \leqslant \Gamma'_k \leqslant n,$$

where $\Gamma'_k = 0$ and $\Gamma'_k = n$ correspond to the nominal and worst cases, respectively. Note that while Γ_i controls the robustness of the i th constraint, Γ'_k controls the robustness of the k th objective function against the level of conservatism. For notational convenience, let $\boldsymbol{\Gamma} \in R^m$ and $\boldsymbol{\Gamma}' \in R^K$ be the vectors of budgets of uncertainty for the constraints and for the objective functions, respectively. Imposing the budgets of uncertainty for the constraints and the objective functions will ensure that the solution will remain both constraint robust and objective function robust. The nonlinear robust formulation of the nominal model in (1) is stated as

equation(11)

$$\begin{aligned} \min \quad & z_k = \max_{\mathbf{c}_k}\left[\bar{f}_k(\mathbf{c}_k, \mathbf{x}) \middle| \sum_{j=1}^{n}\delta'_{kj} \leqslant \Gamma'_k\right] \quad \forall k \\ \text{s.t.} \quad & \max_{\mathbf{a}_i}\left[\bar{g}_i(\mathbf{a}_i, \mathbf{x}) \middle| \sum_{j=1}^{n}\delta_{ij} \leqslant \Gamma_i\right] \leqslant b_i \quad \forall i \\ & \mathbf{x} \in \mathbb{B}^n. \end{aligned}$$

Unlike the single objective model (7), each c_{kj} in the objective functions is considered imprecise in (11).

Any feasible solution to the above model is called a robust feasible solution. The set of all robust feasible solutions, *i.e.*, $X^{\boldsymbol{\Gamma}} = \left\{\mathbf{x} \in \mathbb{B}^n | \max_{\mathbf{a}_i}\left[\bar{g}_i(\mathbf{a}_i, \mathbf{x}) \middle| \sum_{j=1}^{n}\delta_{ij} \leqslant \Gamma_i\right] \leqslant b_i \ \ \forall i\right\}$, is called the robust feasible region in decision space for a given $\boldsymbol{\Gamma}$. A $\mathbf{x} \in X^{\boldsymbol{\Gamma}}$ is called a robust feasible solution in decision space. For given $\boldsymbol{\Gamma}'$ and $\boldsymbol{\Gamma}$, the set $Z^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'} = \left\{\mathbf{z} \in \Re^K | z_k = \max_{\mathbf{c}_k}\left[\bar{f}_k(\mathbf{c}_k, \mathbf{x}) \middle| \sum_{j=1}^{n}\delta'_{kj} \leqslant \Gamma'_k\right], \mathbf{x} \in X^{\boldsymbol{\Gamma}}\right\}$ is the robust feasible region in criterion space. A $\mathbf{z} \in Z^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'}$ is called a robust feasible solution in criterion space or a robust criterion vector. A nondominated robust criterion vector $\bar{\mathbf{z}} \in Z^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'}$, an efficient robust solution $\bar{\mathbf{x}} \in X^{\boldsymbol{\Gamma}}$, and an optimal robust criterion vector $\hat{\mathbf{z}} \in Z^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'}$ can be defined in similar ways to their counterparts for the nominal model (1). Similarly, the robust ideal point $\mathbf{z}^* \in R^K$ is defined as $z^*_k = \min_{\mathbf{x}}\left\{\max_{\mathbf{c}_k}\left[\bar{f}_k(\mathbf{c}_k, \mathbf{x}) \middle| \sum_{j=1}^{n}\delta'_{kj} \leqslant \Gamma'_k\right], \mathbf{x} \in X^{\boldsymbol{\Gamma}}\right\}$ for all k. Corollary 1 in Appendix A provides a formulation to determine $\mathbf{z}^*$. A robust utopian point is also defined as $\mathbf{z}^{**} \in R^K$ such that $z^{**}_k = z^*_k - \varepsilon_k$ with $\varepsilon_k > 0$ and small for all k.

For a given weighting vector $\mathbf{w} \in \mathbf{W}$, a robust augmented weighted Tchebycheff program for the nonlinear programming model in (11) is formulated from (3) as the following

equation(12)

$$
\begin{aligned}
\min \quad & \alpha + \rho \sum_{k=1}^{K} (z_k - z_k^{**}) & \\
\text{s.t.} \quad & \alpha \geqslant w_k (z_k - z_k^{**}) & \forall k \\
& z_k = \max_{\mathbf{c}_k} \left[\bar{f}_k(\mathbf{c}_k, \mathbf{x}) \middle| \sum_{j=1}^{n} \delta'_{kj} \leqslant \Gamma'_k \right] & \forall k \\
& \max_{\mathbf{a}_i} \left[\bar{g}_i(\mathbf{a}_i, \mathbf{x}) \middle| \sum_{j=1}^{n} \delta_{ij} \leqslant \Gamma_i \right] \leqslant b_i & \forall i \\
& x_j \in \{0, 1\} & \forall j \\
& z_k \ \text{unrestricted} & \forall k \\
& \alpha \geqslant 0. &
\end{aligned}
$$

Similar to the coefficients in the objective function of model (7), the coefficients in the objective function of model (12) are exactly known.

An optimal solution to (12) minimizes the augmented weighted Tchebycheff metric between $\mathbf{z}^{**}$ and any $\mathbf{z} \in Z_{\Gamma, \Gamma'}$ while respecting the budget of uncertainty constraints. The solution to this formulation has some interesting properties. First, it is a nondominated solution for the selected Γ and Γ'. Second, unlike its nominal counterpart, it is robust, *i.e.*, insensitive to existing uncertainties in both the objective functions and constraints. This means that given all possible realizations of a_{ij} and c_{kj}, the solution of (12) not only will have a much higher probability of being feasible than the nominal solution of (3) but also will have a corresponding criterion vector which performs comparably well to the nondominated nominal criterion vector. These properties are significant because model (12) can assist the DM as a tool in finding nondominated robust solutions by properly balancing performance versus robustness. Using this formulation, the nominal solution closest to the nominal utopian point $\mathbf{z}^{**}$, measured by the augmented weighted Tchebycheff metric, is slightly sacrificed but, in return, this sacrifice is compensated by the robustness of the solution.

Corollary 2.

Model (12) *has the following mixed binary integer programming counterpart*

equation(13)

$$
\begin{aligned}
\min \quad & d \\
\text{s.t.} \quad & \alpha + \rho \sum_{k=1}^{K} \alpha_k - d \leqslant 0 \\
& w_k \alpha_k - \alpha \leqslant 0 && \forall k \\
& \sum_{j=1}^{n} \bar{c}_{kj} x_j + \Gamma'_k q'_k + \sum_{j=1}^{n} r'_{kj} - \alpha_k \leqslant z_k^{**} && \forall k \\
& \sum_{j=1}^{n} \bar{a}_{ij} x_j + \Gamma_i q_i + \sum_{j=1}^{n} r_{ij} \leqslant b_i && \forall i \\
& q'_k + r'_{kj} \geqslant \hat{c}_{kj} y_j && \forall k, j \\
& q_i + r_{ij} \geqslant \hat{a}_{ij} y_j && \forall i, j \\
& x_j \leqslant y_j, y_j \geqslant 0, x_j \in \{0, 1\} && \forall j \\
& q_i \geqslant 0 && \forall i \\
& \alpha_k, q'_k \geqslant 0 && \forall k \\
& r'_{kj} \geqslant 0 && \forall k, j \\
& r_{ij} \geqslant 0 && \forall i, j \\
& d, \alpha \geqslant 0.
\end{aligned}
$$

Proof.

See Appendix B. □

5. The interactive robust weighted Tchebycheff procedure

In recent years, many interactive methods have been proposed to solve the nominal model (1) of the multiobjective project portfolio selection problem. In this study, the interactive weighted Tchebycheff procedure (Steuer and Choo, 1983 and Steuer, 1986) is used to solve the robust version (11) of the multiobjective project portfolio selection problem. The interactive weighted Tchebycheff procedure is a weighting vector space reduction method (Steuer, 1986) in which the weighting vectors are generated from progressively reduced subsets of **W** defined in (2).

The mixed binary integer programming counterpart (13) of the robust augmented weighted Tchebycheff program (12) is used to generate nondominated robust solutions. Within the interactive weighted Tchebycheff procedure, a set of dispersed weighting vectors is generated at each iteration. The set is then filtered to reduce to a smaller manageable subset. The mixed binary integer programming model in (13) is then solved for each weighting vector in this smaller subset to obtain a set of nondominated criterion vectors. The resulting nondominated robust criterion vectors are then filtered to obtain a smaller subset of dispersed ones. This subset is presented to the DM who selects the most preferred robust solution. In the next iteration, the weighting vector space is reduced around the weighting vector corresponding to the current most preferred solution selected by the DM, new dispersed weighting vectors are generated in this reduced weighting vector space, the newly generated set of weighting vectors is filtered, new nondominated robust solutions are generated, and so on. The procedure terminates after a predetermined number of iterations have been performed or when the DM is satisfied with a nondominated robust solution that has already been found.

In the following, the integer I represents the iteration number. While indicating superscripts in $\boldsymbol{W}^{(I)}$, $\boldsymbol{w}^{(I)}$ and $\mathbf{z}^{(I)}$, I denotes power in r^I. The interactive robust weighted Tchebycheff procedure (Steuer and Choo, 1983 and Steuer, 1986) described step-by-step in the following is similar to that in Drinka, Sun, and Murray (1996).

Step 1.

Determine the maximum number of iterations $I^{\max}$, the number of solutions P to be presented to the DM at each iteration, and the weighting vector space reduction factor r. Let $I = 0$ and $[l_k, u_k] = [0, 1]$ for all k . Obtain z_k^* by solving the robust model (B.1) for each k and then determine the robust utopian point $\mathbf{z}^{**}$.

Step 2.

Let $I = I + 1$. From $\boldsymbol{W}^{(I)} = \left\{\boldsymbol{w} \in \Re^K | w_k \in (l_k, u_k), \sum_{k=1}^{K} w_k = 1\right\}$ randomly generate $20K$weighting vectors. Reduce the $20K$ weighting vectors to obtain the $2P$ most widely dispersed ones.

Step 3.

Solve the robust mixed binary integer programming model (13) for each weighting vector to obtain $2P$nondominated robust criterion vectors. Reduce the $2P$ nondominated robust criterion vectors to Pmost dispersed ones.

Step 4.

Present the P nondominated robust criterion vectors, together with the most preferred solution from the previous iteration $\mathbf{z}^{(I-1)}$ if $I > 1$, to the DM to acquire the most preferred solution $\mathbf{z}^{(I)}$ at iteration I.

Step 5.

Terminate the solution process if $I = I^{\max}$ or if the DM is satisfied with $\mathbf{z}^{(I)}$; continue otherwise.

Step 6.

Compute the weighting vector $\mathbf{w}^{(I)}$ that can generate the current most preferred solution $\mathbf{z}^{(I)}$ with

equation(14)

$$w_k^{(l)} = \frac{\left(z_k^{(l)} - z_k^{**}\right)^{-1}}{\sum_{k'=1}^{K}\left(z_{k'}^{(l)} - z_{k'}^{**}\right)^{-1}},$$

and update l_k and u_k using (15) in the following

equation(15)

$$[l_k, u_k] = \begin{cases} [0, r^l], & w_k^{(l)} - r^l/2 \leqslant 0 \\ [1 - r^l, 1], & w_k^{(l)} - r^l/2 \geqslant 1 \\ [w_k^{(l)} - r^l/2, w_k^{(l)} + r^l/2], & \text{Otherwise.} \end{cases}$$

Go to Step 2.

Note that when $\Gamma = 0$ and $\Gamma' = 0$, the above procedure reduces to the interactive weighted Tchebycheff procedure for the nominal problem (1). The size of 20*K* weighting vectors in Step 2 is a generally accepted size in order to generate widely dispersed weighting vectors (Steuer, 1986). Similar to Steuer (2003), the relative distance measure is used to reduce the 20*K* weighting vectors to 2*P* in Step 2, and to reduce the 2*P*robust criterion vectors to *P* in Step 3. Appendix C has a discussion on the approach used to reduce the set of vectors.

The choices of I^{max} and *P* are dependent on DM's preferences. Larger values for I^{max} increase the decision making time and the burden on the DM but increase solution quality, whereas larger values of *P* make the comparison of multiple solutions more time consuming and increase the burden on the DM but may elicit more preference information from the DM.
Both I^{max} and *P* may be revised during the solution process upon DM's desire. For a detailed discussion of the interactive weighted Tchebycheff procedure, see Steuer and Choo (1983) and Steuer (1986).

Value functions are usually used as proxy DMs in computational experiments to test the performance of solution procedures. $_{LP}$-metric value functions are used to act as proxy DMs in this study in order to simulate the solution process using the interactive robust weighted Tchebycheff procedure with the involvement of a DM. The $_{LP}$-metric value function of a criterion vector $\mathbf{z} \in R^K$ has the following functional form

equation(16)

$$V_{\mathcal{P}}(\mathbf{z}) = \mathcal{K} - \left[\sum_{k=1}^{K} [w'_k(z_k - z_k^{**})]^{\mathcal{P}}\right]^{\frac{1}{\mathcal{P}}},$$

where K is a large constant ensuring $_{VP}(\mathbf{z})>0$ for all feasible robust criterion vectors, $\mathbf{w}'\in\mathbf{W}$ is a weighting vector selected by the user for the purpose of computational experiment, and P⩾1 is an integer. The value function $_{VP}(\mathbf{z})$ in (16), however, is used only as a proxy DM to evaluate representative solutions in Step 4 of the interactive robust weighted Tchebycheff procedure and is not used directly in searching for the optimal solution in the solution process.

6. An illustrative example

A project portfolio selection problem presented in Santhanam and Kyparisis (1995) is used to demonstrate the proposed interactive robust Tchebycheff procedure. Ringuest and Graves (2000) also used the same problem to test their solution method.

An IT company faces the selection of a portfolio from a total of n = 14 projects where data on costs, benefits, and other related information for these projects are estimated. Existing cost interdependencies and synergistic benefits among projects are also identified. There are m = 2 resource constraints for hardware costs and software costs of the projects that must be satisfied. The problem has K = 3 objectives: maximization of total benefits, minimization of total risk scores, and minimization of total miscellaneous costs. The total hardware and software budgets are 20,000 and 6000, respectively. Table 1 and Table 2 present the original problem data.

Table 1. Original estimates for independent benefits, costs, and risk scores.

Project	Mandated	Contingent upon	Annual benefits	Hardware costs	Software costs	Miscellaneous costs	Risk scores
1	Yes	-	1600	16,000	3250	0	5
2	No	1	425	500	1000	0	4
3	No	2	213	350	350	0	3
4	No	2	213	500	500	0	3
5	No	-	2600	2500	2500	0	3
6	No	-	750	1000	1000	0	3
7	No	5	11	0	28	0	1
8	No	5	11	0	27	0	1
9	No	5	3	0	7	0	1
10	No	5	18	0	44	0	1
11	No	1	40,800	0	0	10,200	2
12	No	11	1200	0	0	300	0
13	No	11	3000	0	0	750	1
14	No	11	8000	0	0	2000	0

Table 2. Original estimates for interdependent costs and benefits.

Interdependent projects	Additional benefits	Shared hardware costs	Shared software costs
2, 3			155

2, 4			225
3, 4	85	268	188
4, 5		350	200
4, 6		250	175
5, 6		250	125
12, 13, 14	3400		
4, 5, 6		600 †	375 †

† Original values changed to match the formulation of Appendix D.

Similar to those in Santhanam and Kyparisis (1995), 22 binary variables (x_j) are defined and used to model the selection of a portfolio from the n = 14 projects as well as to model the 8 project interdependencies. The final linearized multiobjective binary integer programming model is formulated as model (D.1) in Appendix D. Note that maximization of total benefits is treated as a minimization objective function in model (D.1). However, the corresponding positive values of this objective function are reported in the tables and texts. The ideal solution is $\mathbf{z}^*$ = (60643, 5, 0). For this illustrative example, $\varepsilon_k = 0$, $\forall k$, is used, hence $\mathbf{z}^{**} = \mathbf{z}^*$.

In the following, a large set of nondominated solutions is generated first for the deterministic, *i.e.*, the nominal, model by solving augmented weighted Tchebycheff programs (3). The interactive weighted Tchebycheff procedure is then applied to the nominal model. The interval uncertainties on coefficients in the objective functions and constraints are considered next and the interactive robust weighted Tchebycheff procedure is applied to find preferred robust solutions for proxy DMs represented by the $_{\mathrm{LP}}$-metric value functions (16). A simulation study is finally performed to evaluate the feasibility and quality of the solutions by introducing uncertainties into the coefficients. The parameters used in the $_{\mathrm{LP}}$-metric value function (16) are arbitrarily set to K = 20, 000 and $\mathbf{w}'$ = (0.3, 0.4, 0.3) for this example. In the interactive robust weighted Tchebycheff procedure, $I^{\max} = 8$, $P = 8$, and $r = 0.2$ are used. In the following tables reporting results, the headings R&G, S&K, and H&N&S represent the results from Ringuest and Graves, 2000 and Santhanam and Kyparisis, 1995, and the current study, respectively.

All computations were conducted on a personal computer with a 2 GHz Core 2 Duo processor and 3 GB of RAM. The reported results reflect the performance of the proposed approach on this computer.

6.1. Generation of nondominated solutions

The augmented weighted Tchebycheff program (3) formulated from model (D.1) is used to generate nondominated solutions for the nominal model (1). Generation of a large set of nondominated solutions is usually the first phase in solving multiobjective programming problems requiring *a posteriori* articulation of the DM’s preference information (Hwang & Masud, 1979). A set of 300 weighting vectors is randomly generated from **W** defined in (2) that is then reduced to a subset of 200 most dissimilar ones. An augmented weighted Tchebycheff program is solved for each of the 200 weighting vectors. Because some augmented weighted Tchebycheff programs formulated with different weighting vectors share the same optimal solution, only 39 distinct nondominated criterion vectors are obtained. The solution process took a total CPU time of 190 seconds.

Santhanam and Kyparisis (1995) solved this problem with a goal programming approach with preemptive priorities. They considered two different priorities with different goal targets and solved 25 problems which yielded 8 nondominated and 1 dominated solutions. Ringuest and Graves (2000) solved the same problem using the Parameter Space Investigation (PSI) method (Steuer & Sun, 1995). Using the PSI method, they randomly generated binary solutions over a hyperrectangle that completely encloses the feasible region. These binary solutions were then evaluated by each of the constraints to check for feasibility. Those not satisfying at least one of the constraints were discarded until 100 feasible solutions were generated. The total number of solutions, including feasible and infeasible, was not reported. The solution set was then screened for dominance which resulted in 33 distinct solutions among which 31 are not dominated by others. Table 3 lists all solutions reported by Ringuest and Graves, 2000 and Santhanam and Kyparisis, 1995, and those generated by solving augmented weighted Tchebycheff programs in this study. A solution is represented by the index set of the projects selected into the portfolio, *i.e.*, $J = \{j|x_j = 1\}$.

Table 3. Comparison among solutions found by the three methods.

No.	Benefit	Risk	Cost	Projects selected	S&K	R&G	H&N&S	Status
1	1600	5	0	1			✓	Nondominated
2	4200	8	0	1, 5	✓		✓	Nondominated
3	4218	9	0	1, 5, 10		✓	✓	Nondominated
4	4229	10	0	1, 5, 7, 10		✓	✓	Nondominated
5	4240	11	0	1, 5, 7, 8, 10			✓	Nondominated
6	4243	12	0	1, 5, 7, 8, 9, 10			✓	Nondominated
7	42,400	7	10,200	1, 11	✓		✓	Nondominated
8	43,600	7	10,500	1, 11, 12		✓	✓	Nondominated
9	45,000	10	10,200	1, 5, 11			✓	Nondominated
10	45,018	11	10,200	1, 5, 10, 11			✓	Nondominated
11	45,040	13	10,200	1, 5, 7, 8, 10, 11		✓	✓	Nondominated
12	45,043	14	10,200	1, 5, 7, 8, 9, 10, 11			✓	Nondominated
13	46,200	10	10,500	1, 5, 11, 12			✓	Nondominated
14	46,218	11	10,500	1, 5, 10, 11, 12			✓	Nondominated

15	46,240	13	10,500	1, 5, 7, 8, 10, 11, 12		✓	✓	Nondominated
16	46,243	14	10,500	1, 5, 7, 8, 9, 10, 11, 12			✓	Nondominated
17	46,600	8	11,250	1, 11, 12, 13			✓	Nondominated
18	48,000	11	10,950	1, 5, 11, 13	✓			Nondominated
19	48,040	14	10,950	1, 5, 7, 8, 10, 11, 13		✓	✓	Nondominated
20	48,043	15	10,950	1, 5, 7, 8, 9, 10, 11, 13			✓	Nondominated
21	49,200	11	11,250	1, 5, 11, 12, 13			✓	Nondominated
22	49,240	14	11,250	1, 5, 7, 8, 10, 11, 12, 13			✓	Nondominated
23	49,243	15	11,250	1, 5, 7, 8, 9, 10, 11, 12, 13			✓	Nondominated
24	50,400	7	12,200	1, 11, 14	✓		✓	Nondominated
25	51,600	7	12,500	1, 11, 12, 14	✓		✓	Nondominated
26	53,000	10	12,200	1, 5, 11, 14	✓		✓	Nondominated
27	53,018	11	12,200	1, 5, 10, 11, 14		✓	✓	Nondominated
28	53,029	12	12,200	1, 5, 8, 10, 11, 14		✓		Nondominated
29	53,040	13	12,200	1, 5, 7, 8, 10, 11, 14			✓	Nondominated
30	53,043	14	12,200	1, 5, 7, 8, 9, 10, 11, 14			✓	Nondominated
31	53,400	8	12,950	1, 11, 13, 14			✓	Nondominated
32	54,200	10	12,500	1, 5, 11, 12, 14			✓	Nondominated
33	54,240	13	12,500	1, 5, 7, 8, 10, 11, 12, 14		✓	✓	Nondominated
34	54,243	14	12,500	1, 5, 7, 8, 9, 10, 11, 12, 14			✓	Nondominated
35	56,000	11	12,950	1, 5, 11, 13, 14		✓	✓	Nondominated
36	56,018	12	12,950	1, 5, 10, 11, 13, 14		✓		Nondominated
37	56,029	13	12,950	1, 5, 7 or 8, 10, 11, 13, 14		✓		Nondominated
38	56,040	14	12,950	1, 5, 7, 8, 10, 11, 13, 14			✓	Nondominated
39	56,043	15	12,950	1, 5, 7, 8, 9, 10, 11, 13, 14			✓	Nondominated
40	58,000	8	13,250	1, 11, 12, 13,	✓		✓	Nondominated

				14				
41	60,600	11	13,250	1, 5, 11, 12, 13, 14			✓	Nondominated
42	60,618	12	13,250	1, 5, 10, 11, 12, 13, 14		✓		Nondominated
43	60,629	13	13,250	1, 5, 7, 10, 11, 12, 13, 14		✓		Nondominated
44	60,640	14	13,250	1, 5, 7, 8, 10, 11, 12, 13, 14			✓	Nondominated
45	60,643	15	13,250	1, 5, 7, 8, 9, 10, 11, 12, 13, 14	✓	✓	✓	Nondominated
46	4232	11	0	1, 5, 8, 9, 10		✓		Dominated by No. 5
47	43,150	10	10,200	1, 6, 11		✓		Dominated by No. 9
48	44,350	10	10,500	1, 6, 11, 12		✓		Dominated by No. 13
49	45,022	12	10,200	1, 5, 7, 8, 11		✓		Dominated by 1, 5, 8, 10, 11 (CE)
50	46,214	12	10,500	1, 5, 8, 9, 11, 12		✓		Dominated by No. 14
51	48,011	12	10,950	1, 5, 8, 11, 13		✓		Dominated by 1,5,10,11,13 (CE)
52	48,022	13	10,950	1, 5, 7, 8, 11, 13		✓		Dominated by 1, 5, 8, 10, 11, 13 (CE)
53	48,032	14	10,950	1, 5, 7, 9, 10, 11, 13	✓			Dominated by No. 19
54	49,211	12	11,250	1, 5, 7, 11, 12, 13		✓		Dominated by 1, 5, 10, 11, 12, 13 (CE)
55	49,222	13	11,250	1, 5, 7, 8, 11, 12, 13		✓		Dominated by 1, 5, 8, 10, 11, 12, 13 (CE)
56	49,232	14	11,250	1, 5, 7, 9, 10, 11, 12, 13		✓		Dominated by No. 22
57	52,350	10	12,500	1, 6, 11, 12, 14		✓		Dominated by No 26
58	53,032	13	12,200	1, 5, 7, 9, 10, 11, 14		✓		Dominated by No 29
59	54,203	11	12,500	1, 5, 9, 11, 12, 14		✓		Dominated by 1, 5, 10, 11, 12, 14 (CE)
60	54,222	12	12,500	1, 5, 7, 8, 11, 12, 14		✓		Dominated by 1, 5, 8, 10, 11, 12, 14 (CE)
61	56,032	14	12,950	1, 5, 7 *or* 8, 9, 10, 11, 13, 14		✓		Dominated by No. 38
62	60,632	14	13,250	1, 5, 8, 9, 10, 11, 12, 13, 14		✓		Dominated by No. 44

A complete enumeration (CE) was performed to find all efficient solutions for this problem. The problem has 234 feasible solutions of which 63 are efficient and 171 are not. From the set of efficient solutions, 54 distinct nondominated criterion vectors are found. There is a difference between the number of efficient solutions and the number of nondominated criterion vectors because some different portfolios share the same criterion vectors. Because projects 7 and 8 have the same coefficients in the objective functions, given that the solution remains feasible, replacing one with the other in an efficient portfolio will create a different efficient portfolio with the same criterion vector. A further examination of the 31 solutions not dominated by others reported in Ringuest and Graves (2000) found only 15 of them are actually nondominated.

The total number of feasible solutions, number of nondominated solutions and the number of nondominated criterion vectors found by these methods are summarized in Table 4. Table 4 shows that the augmented weighted Tchebycheff program can generate a richer set of nondominated criterion vectors than the other methods with similar computational efforts. In addition, solutions generated with the augmented weighted Tchebycheff programs are guaranteed to be nondominated whereas those found with the other two methods may be dominated.

Table 4. Performance summary of the three methods.

Method	Feasible solutions	Nondominated solutions	Nondominated criterion vectors
Santhanam and Kyparisis (S&K)	9	8	8
Ringuest and Graves (R&G)	100	16	15
Augmented weighted Tchebycheff (H&N&S)	39	39	39
Complete enumeration (CE)	234	63	54

6.2. Interactive weighted Tchebycheff procedure applied to the nominal model

The interactive weighted Tchebycheff procedure is applied to the nominal model to search for a final solution for proxy DMs represented by the $_{LP}$-metric value functions (16). For the nominal model, $\Gamma'_k = 0$for k = 1, 2, 3 and Γ_i = 0 for i = 1, 2.

Because all nondominated solutions are found through CE, the optimal solution for each proxy DM can be found by directly evaluating all nondominated solutions with the corresponding value function and then selecting the solution with the largest value. These optimal solutions are then used to measure the quality of the solutions obtained with different methods. Solutions obtained with the interactive weighted Tchebycheff procedure at successive iterations as well as the optimal solutions obtained by R&G, S&K, and CE are summarized in Table 5, Table 6 and Table 7 for the $_{LP}$-metric value functions with P=2, P=4and P=∞, respectively. As it is clear from these tables, the interactive weighted Tchebycheff procedure can find the optimal solution within 3 or 4 iterations. In addition, although Ringuest and Graves (2000) and Santhanam and Kyparisis

(1995) did not use any value function as proxy DM to search for the optimal solution, their best solutions for the L_P-metric value functions with P=2, P=4 and P=∞ are close to the optimal solutions.

Table 5. Deterministic solutions for the L_2-metric value function.

Source	Selected solution	Benefit	Risk	Cost	V_2 (**z**)
H&N&S Iter. 1‑2	1, 5, 11, 12, 14	54,200	10	12,500	15,781.1605
H&N&S Iter. 3	1, 11, 12, 13, 14	58,000	8	13,250	15,946.6906
H&N&S Iter. 4‑8	1, 5, 7, 8, 10, 11, 12, 13, 14	60,640	14	13,250	16,024.9983
R&G	1, 5, 7, 8, 9, 10, 11, 12, 13, 14	60,643	15	13,250	16,024.9980
S&K	1, 5, 7, 8, 9, 10, 11, 12, 13, 14	60,643	15	13,250	16,024.9980
CE	1, 5, 7, 8, 10, 11, 12, 13, 14	60,640	14	13,250	16,024.9983

Table 6. Deterministic solutions for the L_4-metric value function.

Source	Selected solution	Benefit	Risk	Cost	V_4 (**z**)
H&N&S Iter. 1	1, 11, 12, 14	51,600	7	12,500	16,016.0299
H&N&S Iter. 2	1, 5, 11, 12, 14	54,200	10	12,500	16,185.5093
H&N&S Iter. 3‑8	1, 5, 7, 8, 9, 10, 11, 14	53,043	14	12,200	16,209.3660
R&G	1, 5, 7, 9, 10, 11, 14	53,032	13	12,200	16,208.6466
S&K	1, 5, 11, 14	53,000	10	12,200	16,206.5382
CE	1, 5, 7, 8, 9, 10, 11, 14	53,043	14	12,200	16,209.3660

Table 7. Deterministic solutions for the L_∞-metric value function.

Source	Selected solution	Benefit	Risk	Cost	V_∞ (**z**)
H&N&S Iter. 1	1, 11, 12, 14	51,600	7	12,500	16,250.0000
H&N&S Iter. 2	1, 5, 11, 12, 14	54,200	10	12,500	16,250.0000
H&N&S Iter. 3	1, 11, 14	50,400	7	12,200	16,340.0000
H&N&S Iter. 4‑8	1, 5, 7, 8, 9, 10, 11, 12, 13	49,243	15	11,250	16,580.0000
R&G	1, 5, 7, 9, 10, 11, 12, 13	49,232	14	11,250	16,576.7000
S&K	1, 11, 14	50,400	7	12,200	16,340.0000
CE	1, 5, 7, 8, 9, 10, 11, 12, 13	49,243	15	11,250	16,580.0000

6.3. Interactive robust weighted Tchebycheff procedure applied to the robust model

In the following, the data in Table 1 and Table 2 are viewed as nominal values and interval uncertainties are introduced into all costs, benefits, and risk scores. Table 8 and Table 9 present the half-interval widths for the data in Table 1 and Table 2, respectively.

Table 8. Half-interval widths for independent benefits, costs, and risk scores.

Project	Annual benefits	Hardware costs	Software costs	Miscellaneous costs	Risk scores
1	320	2575	710	0	1.25
2	42.5	150	350	0	0.6
3	10.65	105	122.5	0	0.3
4	10.65	150	175	0	0.3
5	260	750	875	0	0.3
6	75	300	350	0	0.3
7	1.1	0	9.8	0	0.05
8	1.1	0	9.45	0	0.05
9	0.09	0	2.45	0	0.1
10	1.8	0	15.4	0	0.1
11	10200	0	0	3060	0.2
12	120	0	0	30	0
13	300	0	0	75	0.1
14	1600	0	0	300	0

Table 9. Half-interval widths for interdependent costs and benefits.

Interdependent projects	Additional benefits	Shared hardware costs	Shared software costs
2, 3			54.25
2, 4			78.75
3, 4	4.25	80.4	65.8
4, 5		105	70
4, 6		75	61.25
5, 6		75	43.75
12, 13, 14	1020		
4, 5, 6		75	43.75

Given all possible outcomes of the uncertainties, the purpose of the solution process is to find a DM's most preferred nondominated solution which is robust in terms of uncertainties in both objective functions and constraints. In the solution process, the mixed binary integer programming counterpart (13) of the robust augmented weighted Tchebycheff program (12) is solved in Step 3 of the iterative robust weighted Tchebycheff procedure. However, in order for the DM to be impartial toward robustness of the presented solutions, the proxy DM is assumed to always choose the preferred solution in Step 4 using the nominal criterion vectors, *i.e.* , the criterion vectors computed with the nominal coefficient values in the objective functions are evaluated with the $_{\mathrm{LP}}$-metric value functions (16). Note that this assumption is made merely to facilitate the illustration of the solution process.

For demonstration purposes, arbitrary values of $\Gamma'_k = 0.7$ for k = 1, 2, 3 and Γ_i = 0.5, 1.0, 1.5 for i = 1, 2 are used in order to examine the effect of the budget of uncertainty levels on solution feasibility and quality. The interactive procedure starts with lower budget of uncertainty levels, *i.e.*, with $\Gamma_i = 0.5$ for $i = 1, 2$. Similar to the interactive procedure discussed in Section 6.2, the DM is presented with a set of robust, rather than deterministic, solutions at each iteration of the interactive procedure. The DM selects the most preferred solution and decides to either proceed to the next iteration or terminate the solution process. This process is performed for the three levels of robustness, *i.e.*, $\Gamma_i = 0.5, 1.0, 1.5$ for $i = 1, 2$. In order to keep the results concise, only the final solutions obtained with the iterative robust weighted Tchebycheff procedure are presented. The solutions for the L_P-metric value functions with P=2, P=4 and P=∞ are reported in Table 10, Table 11 and Table 12, respectively.

Table 10. Robust solutions for the L_2-metric value function.

Γ_i	Selected solution	Nominal benefit	Nominal risk	Nominal cost	V_2 (**z**)
0.5	1, 6, 11, 12, 13, 14	58,750	11	13,250	15,984.6369
1.0	1, 6, 11, 12, 13, 14	58,750	11	13,250	15,984.6369
1.5	1, 11, 12, 13, 14	58,000	8	13,250	15,946.6906

Table 11. Robust solutions for the L_4-metric value function.

Γ_i	Selected solution	Nominal benefit	Nominal risk	Nominal cost	V_4 (**z**)
0.5	1, 2, 3, 6, 11, 14	51,788	17	12,200	16,108.8844
1.0	1, 6, 11, 12, 14	52,350	10	12,500	16,080.2516
1.5	1, 6, 11, 12, 14	52,350	10	12,500	16,080.2516

Table 12. Robust solutions for L_∞-metric value function.

Γ_i	Selected solution	Nominal benefit	Nominal risk	Nominal cost	V_∞ (**z**)
0.5	1, 2, 3, 4, 11, 14	51,336	17	12,200	16,340.0000
1.0	1, 11, 14	50,400	7	12,200	16,340.0000
1.5	1, 11, 14	50,400	7	12,200	16,340.0000

As expected, the value function is negatively correlated with the budget of uncertainty levels because an increase in the budget of uncertainty generally shrinks the feasible region and may render the current solution infeasible. It is also observed that as consequences of choosing the most preferred robust solution instead of the most preferred nominal solution, the value function deteriorates by at most 0.49%, 0.80%, and 1.45% for the L_P-metric value functions with P=2, P=4 and P=∞, respectively. Therefore, it appears that for each P, the DM's most preferred robust solution is at close proximity of the most preferred nominal solution.

6.4. Simulation study

Since input data are fraught with uncertainty, comparing the performance of solutions using the nominal criterion vectors may not legitimately capture the effects of uncertainty on solution feasibility and quality. To address this issue, a simulation study is performed to mimic the performance of the above nominal and robust solutions in the real world. For real-world applications, the simulation study may be performed at each iteration of the interactive robust weighted Tchebycheff procedure so that the DM can compare the trial solutions using the robust rather than the nominal values of the objective functions. The DM then makes tradeoffs among the objective functions as well as between robustness and performance.

For each imprecise coefficient in the objective functions and in the constraints of the project portfolio selection model, a value is randomly selected from its uncertainty interval. These selected coefficient values, instead of the nominal values, are then used in model (D.1) to formulate the project portfolio selection model. The formulated model is not solved but is used to evaluate the final solutions of the nominal model and of the robust model obtained with the interactive robust weighted Tchebycheff procedure. The best solutions reported in Ringuest and Graves (2000) and Santhanam and Kyparisis (1995) are also evaluated with the formulated model. Each set of randomly selected coefficient values represents one realized instance of the imprecise coefficients in the project portfolio selection model. A total of 10,000 sets of randomly selected coefficient values are generated. Each solution is then evaluated by the constraints of the formulated model to determine if it is feasible. Only when the solution is feasible, it is evaluated with the three objective functions of the formulated model to obtain the corresponding values for each z_k. The criterion vectors **z** are then evaluated by (16) to calculate the corresponding values of$_{VP}$(**z**) whereas z_k^{**} is also updated for the realized coefficient values. The percentage of feasible solutions over the 10,000 realizations of the uncertain coefficients, the average and the worst-case performance of the individual objective functions and $_{VP}$(**z**) for the $_{LP}$-metric value functions with P=2,P=4 and P=∞, are reported in Table 13, Table 14 and Table 15, respectively.

Table 13. Performance for a proxy DM with a L_2-metric value function.

Solution	Source	Feasible (%)	V_2 (**z**)		Benefit		Risk		Cost	
			Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst
1, 5, 7, 8, 10, 11, 12, 13, 14	Nominal	47.20	16,029.2	15,026.5	60,622.3	47,817.5	14.0	15.8	13,236.2	16,578.3
1, 5, 7, 8, 9, 10, 11, 12, 13, 14	R&G/S&K	46.99	16,029.6	15,026.5	60,625.1	47,820.4	15.0	16.8	13,234.5	16,578.3
1, 6, 11, 12,	Γ_i = 0.5,	100.00	15,984.3	14,972.8	58,760.0	46,073.9	11.0	12.7	13,247.8	16,636.2

13, 14	1.0									
1, 11, 12, 13, 14	$\Gamma_i = 1.5$	100.00	15,945.7	14,937.6	58,010.4	45,359.9	8.0	9.5	13,247.8	16,636.2

Table 14. Performance for a proxy DM with a L_4-metric value function.

Solution	Source	Feasible (%)	V_4 ($\mathbf{z}$)		Benefit		Risk		Cost	
			Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst
1, 5, 7, 8, 9, 10, 11, 14	Nominal	46.99	16,194.3	15,259.8	53,023.0	40,976.6	14.0	15.8	12,185.0	15,531.5
1, 5, 7, 9, 10, 11, 14	R&G	48.02	16,192.8	15,259.4	53,023.2	40,965.8	13.0	14.8	12,187.2	15,531.5
1, 5, 11, 14	S&K	51.65	16,191.6	15,257.9	53,003.6	40,934.7	10.0	11.7	12,182.2	15,531.5
1, 2, 3, 6, 11, 14	$\Gamma_i = 0.5$	78.26	16,078.8	15,181.0	51,799.3	39,892.1	17.0	19.4	12,209.2	15,539.2
1, 6, 11, 12, 14	$\Gamma_i = 1.0, 1.5$	100.00	16,058.6	15,129.3	52,353.0	40,535.3	10.0	11.7	12,497.9	15,825.7

Table 15. Performance for a proxy DM with a L_∞-metric value function.

Solution	Source	Feasible (%)	V_∞ ($\mathbf{z}$)		Benefit		Risk		Cost	
			Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst
1, 5, 7, 8, 9, 10, 11, 12, 13	Nominal	46.99	16,343.5	15,687.1	49,220.1	38,473.4	15.0	16.8	11,233.9	14,376.4
1, 5, 7, 9, 10, 11, 12, 13	R&G	48.02	16,341.2	15,687.1	49,220.0	38,463.1	14.0	15.9	11,235.9	14,376.4
1, 11, 14	S&K	100.00	16,299.9	15,338.2	50,403.2	38,494.5	7.0	8.4	12,198.1	15,539.2
1, 2, 3, 4, 11, 14	$\Gamma_i = 0.5$	99.99	16,332.7	15,338.2	51,340.0	39,455.2	17.0	19.4	12,198.2	15,539.2

1, 11, 14	Γ_i = 1.0, 1.5	100.00	16,299.9	15,338.2	50,403.2	38,494.5	7.0	8.4	12,198.1	15,539.2

From these tables, it is first observed that non-robust solutions have generally very low percentages of feasibility. On the other hand, the robust solutions have generally very high percentage of feasibility. This is a critical issue because the feasibility of a solution is always the primary concern of any optimization problem. In Table 13, Table 14 and Table 15, results for any budget of uncertainty higher than 1.5 are not reported because the feasibility of solutions is always satisfied when Γ_i = 1.5 and hence, further increase in Γ_i may deteriorate the quality of the solutions without the need to improve feasibility. In addition, for the L_2-metric value function in Table 13, 100% of the solutions are feasible when Γ_i = 0.5. Hence, the solutions obtained for $\Gamma_i > 0.5$ are reported in this table just for completeness.

Table 16 presents the performance of the robust solutions for the individual objective functions and the L_P-metric value functions with P=2, P=4 and P=∞. In the table, the percentages of change in feasibility, in the value functions, and in the individual objective functions are calculated with respect to the corresponding optimal nominal solutions.

Table 16. Change in performance (%) using robust rather than nominal solutions.

P	Robust solution	Feasible	$v_P(\mathbf{z})$		Benefit		Risk		Cost	
			Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst
2	1, 6, 11, 12, 13, 14	52.80	-0.28	-0.36	-3.07	-3.65	-21.50	-19.16	0.09	0.35
2	1, 11, 12, 13, 14	52.80	-0.52	-0.59	-4.31	-5.14	-42.92	-39.80	0.09	0.35
4	1, 2, 3, 6, 11, 14	31.27	-0.71	-0.52	-2.31	-2.65	21.38	22.69	0.20	0.05
4	1, 6, 11, 12, 14	53.01	-0.84	-0.86	-1.26	-1.08	-28.64	-25.88	2.57	1.89
∞	1, 2, 3, 4, 11, 14	53.00	-0.07	-2.22	4.31	2.55	13.27	15.07	8.58	8.09
∞	1, 11, 14	53.01	-0.27	-2.22	2.40	0.05	-53.39	-49.90	8.58	8.09

As previously mentioned, a table similar to Table 16, without the columns labeled $v_P(\mathbf{z})$, may be presented to the DM at each iteration of the interactive robust weighted Tchebycheff procedure. Each row of the table will represent one trial solution including the DM's most preferred solution selected in the previous iteration if $I > 1$. The DM then makes tradeoffs among the robust objective function values and between robustness and performance when choosing the current most preferred solution.

7. Conclusions

In this study, the problem of selecting a portfolio of R&D projects is considered when there are multiple conflicting objectives and when there are uncertainties in problem data including objective function and constraint coefficients. A robust augmented weighted Tchebycheff program is formulated and its linear counterpart is employed within the interactive robust weighted Tchebycheff procedure to generate robust nondominated solutions. The final portfolio is most preferred by the DM and is robust in terms of all possible realizations of imprecise problem coefficients. Through an illustrative example, the robust solutions are shown to have not only a very high percentage of feasibility but also average and worst-case performance that is comparable to that of the nominal solutions.

An extraordinary strength of the proposed approach is that this robustness is achieved without bothering the DM in supplying unknown distribution details for the imprecise coefficients which is a major inconvenience in practical applications. Moreover, the proposed approach can be readily extended to other multiobjective mixed integer programming problems with uncertainties existing in both objective function and constraint coefficients. Extending ellipsoidal uncertainty to multiobjective programming problems should also serve as a direction for future research.

Appendix A. Determining the robust ideal point

Corollary 1.

The robust ideal point $\mathbf{z}^*$ *can be determined from the following model for all* $k = 1, \cdots, K$,

equation(A.1)

$$
\begin{aligned}
\min \quad & z_k \\
\text{s.t.} \quad & \sum_j \bar{c}_{kj} x_j + \Gamma'_k q'_k + \sum_j r'_{kj} \leqslant z_k && \forall k \\
& \sum_j \bar{a}_{ij} x_j + \Gamma_i q_i + \sum_j r_{ij} \leqslant b_i && \forall i \\
& q'_k + r'_{kj} \geqslant \hat{c}_{kj} y_j && \forall k, j \\
& q_i + r_{ij} \geqslant \hat{a}_{ij} y_j && \forall i, j \\
& -y_j \leqslant x_j \leqslant y_j, x_j \in (0,1), y_j \geqslant 0 && \forall j \\
& q_i \geqslant 0 && \forall i \\
& q'_k \geqslant 0, \quad z_k \text{ unrestricted} && \forall k \\
& r'_{kj} \geqslant 0 && \forall k, j \\
& r_{ij} \geqslant 0 && \forall i, j.
\end{aligned}
$$

Proof.

The robust ideal point $\mathbf{z}^*$ can be determined using the following nonlinear model for all $k = 1, \cdots, K$,

equation(A.2)

$$
\begin{aligned}
\min \quad & z_k = \max_{\mathbf{c}_k} \left[\bar{f}_k(\mathbf{c}_k, \mathbf{x}) | \sum_j \delta'_{kj} \leqslant \Gamma'_k \right] \\
\text{s.t.} \quad & \max_{\mathbf{a}_i} \left[\bar{g}_i(\mathbf{a}_i, \mathbf{x}) | \sum_j \delta_{ij} \leqslant \Gamma_i \right] \leqslant b_i \quad \forall i \\
& x_j \in \{0, 1\} \quad \forall j.
\end{aligned}
$$

Model (A.2) can be reformulated as model (A.3) in the following

equation(A.3)

$$\begin{aligned} \min \quad & z_k \\ \text{s.t.} \quad & \max_{\mathbf{c}_k}\left[\bar{f}_k(\mathbf{c}_k,\mathbf{x}) \middle| \sum_j \delta'_{kj} \leqslant \Gamma'_k\right] \leqslant z_k \quad \forall k \\ & \max_{\mathbf{a}_i}[\bar{g}_i(\mathbf{a}_i,\mathbf{x}) | \sum_j \delta_{ij} \leqslant \Gamma_i\Big] \leqslant b_i \quad \forall i \\ & x_j \in \{0,1\} \quad \forall j \\ & z_k \text{ unrestricted} \quad \forall k \end{aligned}$$

Model (B.1) is directly obtained from model (A.3) by following the derivation of (8) from (7). □

Appendix B. Proof of Corollary 2

Model (12) is first reformulated as

equation(B.1)

$$\begin{aligned} \min \quad & d \\ \text{s.t.} \quad & d \geqslant \alpha + \rho \sum_k \alpha_k \quad \forall k \\ & \alpha \geqslant w_k \alpha_k \quad \forall k \\ & \max_{\mathbf{c_k}}\left[\bar{f}_k(\mathbf{c}_k,\mathbf{x}) \middle| \sum_j \delta'_{kj} \leqslant \Gamma'_k\right] - \alpha_k \leqslant z_k^{**} \quad \forall k \\ & \max_{\mathbf{a}_i}\left[\bar{g}_i(\mathbf{a}_i,\mathbf{x}) \middle| \sum_j \delta_{ij} \leqslant \Gamma_i\right] \leqslant b_i \quad \forall i \\ & x_j \in \{0,1\} \quad \forall j. \end{aligned}$$

Using the derivation of (8) from (7) and (13) follows. □

Appendix C. Procedure for filtering a set of vectors

The following procedure reduces 2*P* robust criterion vectors to *P* most dispersed ones using the *d*-norm relative distance measure (Steuer, 2003). In the procedure, Z^{2P} represents the set of the robust criterion vectors that have not been selected while Z^{P} the set of the most dispersed ones that have been selected.

Step 1.

Calculate $\bar{z}_k = \max\{z_k | \mathbf{z} \in Z^{2P}\}$, $\underline{z}_k = \min\{z_k | \mathbf{z} \in Z^{2P}\}$, $\pi_k = (\bar{z}_k - \underline{z}_k)^{-1} \Big/ \sum_k (\bar{z}_k - \underline{z}_k)^{-1}$. Randomly select a vector from Z^{2P} and transfer it to Z^{P}.

Step 2.

Find $\mathbf{z}^{\max} \in Z^{2P}$ so that $\mathbf{z}^{\max}$ is the most dissimilar vector from all vectors in Z^P, that is

$$\min_{\mathbf{z}' \in Z^P}\left[\sum_{k=1}^{K}\left(\pi_k\left|z_k^{\max} - z'_k\right|\right)^d\right]^{1/d} = \max_{\mathbf{z} \in Z^{2P}}\left[\min_{\mathbf{z}' \in Z^P}\left[\sum_{k=1}^{K}\left(\pi_k\left|z_k - z'_k\right|\right)^d\right]^{1/d}\right]$$

Step 1.

Transfer $\mathbf{z}^{\max}$ to Z^P. If $|Z^P| = P$, then terminate; otherwise go to Step 2.

The procedure above is used in Step 3 of the interactive robust weighted Tchebycheff procedure to reduce $2P$ criterion vectors to P most dispersed ones. It is also used in Step 2 to reduce $20K$ weighting vectors to $2P$ most dispersed ones.

Appendix D. The basic formulation for the illustrative example

equation(D.1)

$$\begin{aligned}
\min\quad & -f_1(x) = -1600x_1 - 425x_2 - 213x_3 - 213x_4 - 2600x_5 - 750x_6 \\
& \qquad - 11x_7 - 11x_8 - 3x_9 - 18x_{10} - 40800x_{11} - 1200x_{12} \\
& \qquad - 3000x_{13} - 8000x_{14} - 85x_{3.4} - 3400x_{12.13.14} \\
\min\quad & f_2(x) = 5x_1 + 4x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6 + x_7 + x_8 + x_9 + x_{10} \\
& \qquad + 2x_{11} + x_{13} \\
\min\quad & f_3(x) = 10200x_{11} + 300x_{12} + 750x_{13} + 2000x_{14} \\
\text{s.t.}\quad & 16000x_1 + 500x_2 + 350x_3 + 500x_4 + 2500x_5 + 1000x_6 - 268x_{3.4} \\
& \qquad - 350x_{4.5} - 250x_{4.6} - 250x_{5.6} - 600x_{4.5.6} \leqslant 20000 \\
& 3250x_1 + 1000x_2 + 350x_3 + 500x_4 + 2500x_5 + 1000x_6 \\
& \qquad 28x_7 + 27x_8 + 7x_9 + 44x_{10} - 155x_{2.3} - 225x_{2.4} - 188x_{3.4} - 200x_{4.5} - 175x_{4.6} - 125x_{5.6} - 375x_{4.5.6} \leqslant 6000 \\
& -x_1 + x_2 \leqslant 0,\ -x_2 + x_3 \leqslant 0,\ x_3 - x_4 \leqslant 0, \\
& -x_5 + x_7 \leqslant 0,\ -x_5 + x_8 \leqslant 0,\ -x_5 + x_9 \leqslant 0, \\
& -x_5 + x_{10} \leqslant 0,\ -x_1 + x_{11} \leqslant 0,\ -x_{11} + x_{12} \leqslant 0, \\
& -x_{11} + x_{13} \leqslant 0,\ -x_{11} + x_{14} \leqslant 0,\ x_2 + x_3 - x_{2.3} \leqslant 1, \\
& -x_2 - x_3 + 2x_{2.3} \leqslant 0,\ x_2 + x_4 - x_{2.4} \leqslant 1,\ -x_2 - x_4 + 2x_{2.4} \leqslant 0, \\
& x_3 + x_4 - x_{3.4} \leqslant 1,\ -x_3 - x_4 + 2x_{3.4} \leqslant 0,\ x_4 + x_5 - (x_{4.5} + x_{4.5.6}) \leqslant 1, \\
& -x_4 - x_5 + 2(x_{4.5} + x_{4.5.6}) \leqslant 0,\ x_4 + x_6 - (x_{4.6} + x_{4.5.6}) \leqslant 1, \\
& -x_4 - x_6 + 2(x_{4.6} + x_{4.5.6}) \leqslant 0,\ x_5 + x_6 - (x_{5.6} + x_{4.5.6}) \leqslant 1, \\
& -x_5 - x_6 + 2(x_{5.6} + x_{4.5.6}) \leqslant 0,\ x_4 + x_5 + x_6 - x_{4.5.6} \leqslant 2, \\
& -x_4 - x_5 - x_6 + 3x_{4.5.6} \leqslant 0,\ x_{12} + x_{13} + x_{14} - x_{12.13.14} \leqslant 2, \\
& -x_{12} - x_{13} - x_{14} + 3x_{12.13.14} \leqslant 0, \\
& x_1 = 1 \\
& x_j \in \{0, 1\}, \quad \forall j
\end{aligned}$$

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